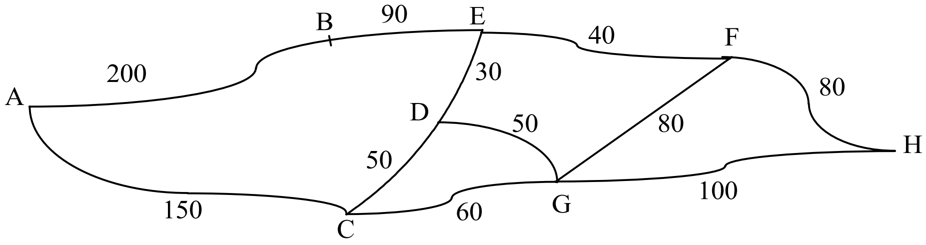
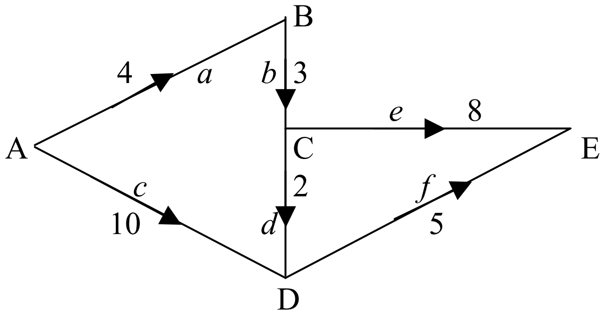
1. A number of cottages on an estate need to be connected together, to provide electricity. Their distances, in metres, from each other are as shown on the map:



Use Kruskal's algorithm to find the minimum spanning tree, and hence the minimum length of cable required. [5]

2. The problem of finding the shortest path between two points on a network can be written as a linear programming problem. For the network shown,



where the length of AB is 4 and a is 1 or 0 depending on whether or not the arc AB is included, (and similarly for the letters on all the other arcs), we get the following system of equations for finding the shortest path from A to E :

Minimise $P = 4a + 3b + 10c + 2d + 8e + 5f$, subject to

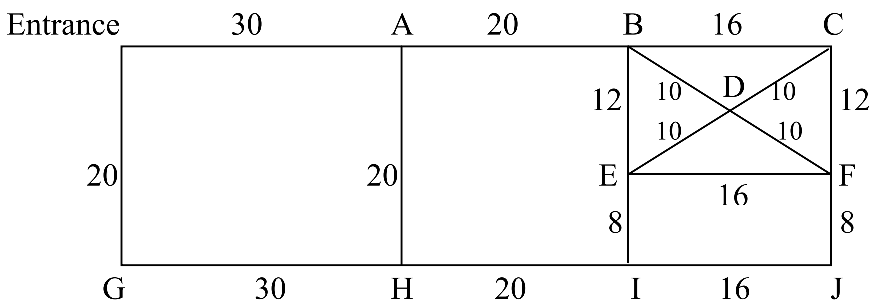
$a + c = 1,\ a - b = 0,\ b - d - e = 0,\ c + d - f = 0,\ f + e = 1;\ a, b, c, d, e, f \geq 0$

A solution is $a = b = d = f = 1$, $c = e = 0$.

(i) State the shortest path from A to E, and give its length. [3]

(ii) Explain how each constraint equation is obtained. [4]

3. A visitor to a museum wants to make sure that she sees every painting, whilst walking the minimum distance. She must start and finish at the entrance. The map of the museum's corridors is shown, with distances given in metres :



(i) Use a suitable algorithm to find the minimum distance she must travel. [5]

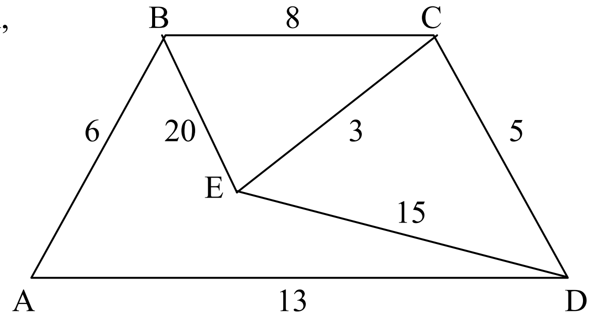
(ii) Give a possible route of this minimum distance. [2]

4. A linear programming problem leads to the initial Simplex tableau :

P	x	y	z	r	s	
1	-2	-5	-4	0	0	0
0	1	3	6	1	0	6
0	2	0	1	0	1	4

(i) Perform one iteration, using the y-column as the pivot column. [3]
(ii) Explain why this does not give the optimal solution for P. [2]
(iii) Write an equation for P in terms of x, z and r, and state which variable should be changed to give a higher value of P. [3]

5. For the weighted graph shown,



(i) find the minimum spanning tree and state its length. [3]
(ii) Starting from A, list all Hamiltonian cycles (paths which pass through every node exactly once). [4]
(iii) Hence write down the solution of the Travelling Salesman Problem for this network. [1]
(iv) It is often stated that twice the minimum spanning tree is an upper bound for the TSP. Explain why that is not true in this example. [3]

6. (i) Use the Shuttle Sort algorithm to sort the following numbers into ascending order :
6 9 4 7 3 8 8 2 5 9 [4]
(ii) Use the first-fit decreasing algorithm to pack these numbers into bins of size 15. [4]
(iii) Prove that it is impossible to pack them into fewer than 5 bins. [2]

7. A car park is to be designed to accommodate cars and lorries. The width of a car is 2 m and that of a lorry is 3.5 m, on average. A strip of 120 m is available to lay out parking bays. A car is charged £1.50 per day, and a lorry £3.50 per day. Local residents, however, have decided that no more than 6 lorries should be allowed to park each day.
(i) Write down two inequalities, apart from $x, y \geq 0$, that must be satisfied by x, the number of cars, and y, the number of lorries. [2]
(ii) Use a graphical method to find the layout that will earn the owner of the car park the most money, assuming that each space is filled by just one vehicle each day. [8]
(iii) The car park owner has the option of getting an additional 1 m of land for a weekly rent of £20. Determine, with a brief explanation, whether this would be worthwhile. [2]

DECISION MATHS 1 (C) PAPER 7 : ANSWERS AND MARK SCHEME

1. Order of selection : DE, EF, {DC and DG (either order)}, FH, EB, CA — M1 M1 A1 A1
   Length of tree = 30 + 40 + 50 + 50 + 80 + 90 + 150 = 490 m — A1 — 5

2. (i) Shortest path is ABCDE, length 4 + 3 + 2 + 5 = 14 — M1 A1 A1
   (ii) Equations equalling 1 e.g. $a + c = 1$, mean that at least one arc must come out of A; equations equalling 0 e.g. $b - d - e = 0$ mean that if an arc into — B2
   a node is included, an arc coming out of that node must also be included — B2 — 7

3. (i) Odd nodes are A, H, I, C; possible pairings are AH + IC = 48, — B1 M1
   AC + IH = 56, AI + CH = 88, so repeat AH and IC, — A1 M1
   giving total distance of 268 + 48 = 316 m — A1
   (ii) e.g. Entrance - G H I J F E D F C B D C D E I E B A H A - Entrance — M1 A1 — 7

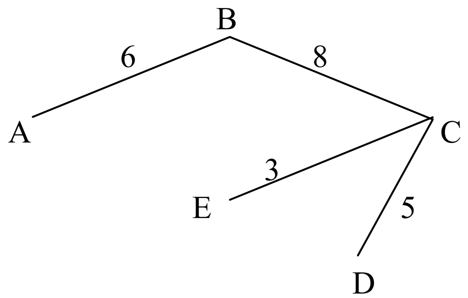
4. (i)

P	x	y	z	r	s	
1	$-^1/_3$	0	6	$1\,^2/_3$	0	10
0	$^1/_3$	1	2	$^1/_3$	0	2
0	2	0	1	0	1	4

M1 A1 A1

   (ii) There are still negative numbers in the top row — B2
   (iii) $P = 10 + {}^1/_3\,x - 6z - 1^2/_3\,r$; increase x to get a greater value for P — B2 B1 — 8

5. (i) M.S.T. :



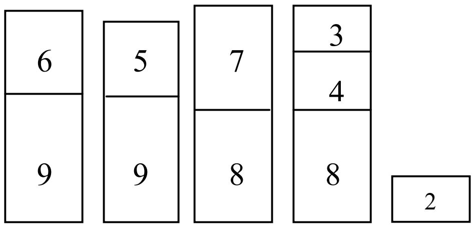
   Length = 22 — M1 A1 A1
   (ii) ABCEDA, ABECDA, ADECBA, ADCEBA — B1 B1 B1 B1
   (iii) Min. path through all nodes once is ABCEDA (or reverse), length 45 — B1
   (iv) Twice M.S.T. is an upper bound only when the graph obeys the triangle inequality i.e. in any triangle formed by three nodes PQR, PQ + QR $\geq$ PR;
   this is not true in this case — B3 — 11

6. (i)

6	9	4	7	3	8	8	2	5	9
4	6	9	7	3	8	8	2	5	9
4	6	7	9	3	8	8	2	5	9
3	4	6	7	9	8	8	2	5	9
3	4	6	7	8	9	8	2	5	9
3	4	6	7	8	8	9	2	5	9
2	3	4	6	7	8	8	9	5	9
2	3	4	5	6	7	8	8	9	9
2	3	4	5	6	7	8	8	9	9

M1 M1 A1 A1

6.. (ii)

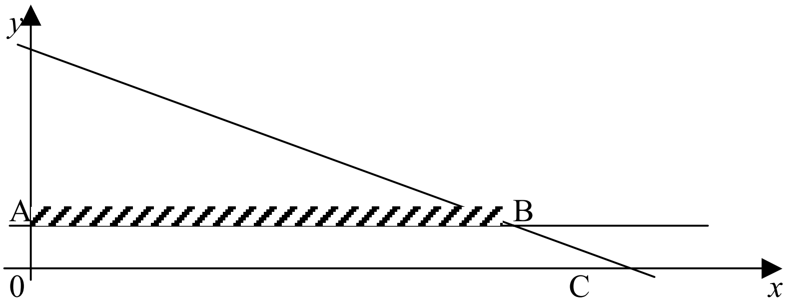


M1 A1
A1 A1

(iii) Total of numbers = 61, so need at least 61 / 15 bins i.e. $4\,{}^{1}/_{15}$, > 4
Therefore need at least 5 bins — M1 A1 — 10

7. (i) $2x + 3.5y \leq 120$ and $y \leq 6$ — B1 B1

(ii) Graphs



B1 B2

$P = 1.5x + 3.5y$
Vertices O (0,0) : Profit = 0, A (0, 6) : P = £21, — B1
B (49.5, 6) : P = £95.25, C (60, 0) : P = £90 — B1 B1
However, cannot have 49.5 cars, so look at neighbouring integer
coordinates : (49, 6) gives P = £94.50 and (50, 5) gives P = £92.50, — M1
so allow 49 cars and 6 lorries. — A1

(iii) This takes up 119 m, leaving 1 m over. An extra 1 m will allow one more car in, giving weekly increased rent of £10.50, so not worth it — M1 A1 — 12